

# A quick algorithm to compute an approximated power spectral density from an arbitrary Allan deviation

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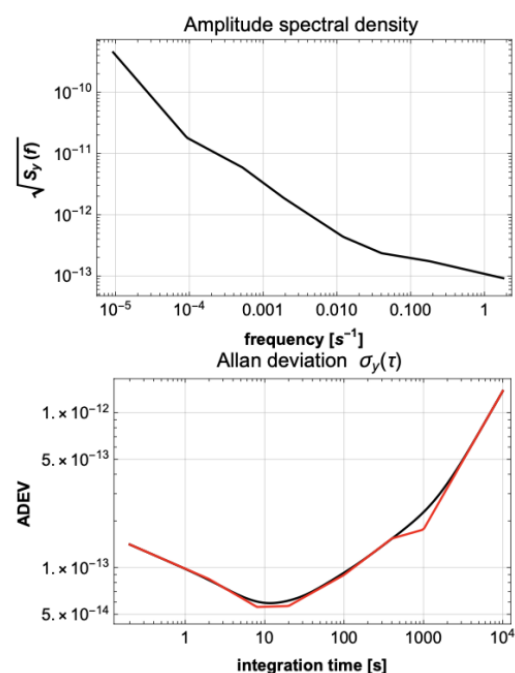
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Oscillator-based devices like clocks, transponders, and synthesizers are vital to wireless communication, digital electronics, and space-based navigation. Evaluating their stability is crucial for assessing the impact of random fluctuations (noise) on system performance. Manufacturers typically specify stability using an Allan/Hadamard Variance (AVAR/HVAR) profile in the time domain. However, the frequency domain provides a more comprehensive description of noise through the Amplitude/Power Spectral Density function (ASD/PSD). Both are second-moment measures of the time series, but it is only possible to translate unambiguously from the PSD to the AVAR/HVAR, not vice versa, except in the case of a single noise type, a very limiting case.

Starting from an arbitrary AVAR/HVAR defined as power-law segments in the time domain, this note outlines an analytical approach to generate an approximated PSD in the corresponding frequency domain. The proposed algorithm is easy to implement, applicable to all noise types and combinations thereof, and self-validated by reconstructing the corresponding AVAR/HVAR by direct calculus. A coupling with well-established algorithms relying on the PSD<sup>2</sup> constitutes a versatile and effective tool for generating multi-colored noise in end-to-end simulations. This is especially relevant in design phases of GNSS, where predictions of the time deviation for free-running clocks help steer the onboard clock selection, as demonstrated in the context of a Lunar Radio Navigation system<sup>3</sup>.



*Testing the algorithm on the Ultra-Stable Oscillator by AccuBeat<sup>1</sup>. Top: ASD obtained by the input ADEV (bottom, red line). Bottom: input ADEV vs. ADEV (black line) re-obtained from the ASD.*

<sup>1</sup> <https://www.accubeat.com/uso>

<sup>2</sup> Timmer, Jens, and Michel Koenig. "On generating power law noise." *Astronomy and Astrophysics*, v. 300, p. 707 300 (1995): 707.

<sup>3</sup>Iess, L. et al. "High Performance Orbit Determination and Time Synchronization for Lunar Radio Navigation Systems." *Proceedings of the 36th International Technical Meeting of the ION GNSS+ 2023*